# Role of Ambiguities and Gauge Invariance in the Calculation of the Radiatively Induced Chern-Simons Shift in Extended Q.E.D.

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We investigate the possibility of Lorentz and CPT violations in the photon sector, of the Chern-Simons form, be induced by radiative corrections arising from the Lorentz and CPT non-invariant fermionic sector of an extended version of QED. By analyzing the modified vacuum polarization tensor, three contributions are considered: two of them can be identified with well known amplitudes; the (identical) QED vacuum polarization tensor and the (closely related) $AVV$ triangular amplitude. These amplitudes are evaluated in their most general form (to include in our discussion automatically the question of ambiguities) on the point of view of a strategy to manipulate and calculate divergent amplitudes that can avoid the explicit calculation of divergent integrals. Rather than this only general properties are used in intermediary steps. With this treatment, the results obtained by others authors can be easily recovered and we show that, if we choose to impose U(1) gauge invariance maintenance in the pure QED calculated amplitudes, to be consistent with the renormalizability, the induced Chern-Simons term assumes a nonvanishing ambiguities free value. However if, in addition, we choose to get an answer consistent with renormalizability by anomaly cancellation of the Standard Model a vanishing value can be obtained, in accordance with what was previously conjectured by other authors.

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## I. INTRODUCTION

The essential ingredient for the construction of Quantum Field Theories are the symmetries. Undoubtedly, the most important of them all is the Lorentz invariance, which represents our conception in respect of space-time. This symmetry has a privileged status when compared to those denominated as internal symmetries. It is fair to say that is lies at the root of practically all success in describing the available phenomenology. In other hand, CPT symmetry has played a crucial hole in the construction of our present theoretical knowledge about the phenomenology of fundamental particle interaction. In particular, for the construction of the standard model Lagrangian, Lorentz and CPT invariance are required in addition to invariance under $SU(3) \otimes SU(2) \otimes U(1)$ gauge transformations [1]. From the point of view of this symmetry content, all possible interactions involving an arbitrary number of fundamental fields, simultaneously invariant under Lorentz, CPT and gauge transformations, are in principle realizations of the model. Other two restrictions reduce drastically the set of interactions suggested by the adopted symmetry content and fixes the number of fundamental quantum fields. The first one is the renormalizability by power counting that eliminates all invariant interaction with canonical dimension $d \succ 4$. The second one is the renormalizability by anomaly cancellation, which forces us to find a number of fermion fields to the theory in such a way the violations in symmetry relations introduced by the anomalies cancel each other to maintain the renormalizability of the model [2]. It is very important to emphasize that these two crucial restrictions above cited do not resides in the symmetry content but are deeply related to the perturbative solution of the theory and therefore to practical reasons. They result directly from our limitations in the treatment of divergences emerging in the perturbative approach. In this context the success of the resulting theory in the accurate description of the corresponding phenomenology is a direct consequence of the validity of the gauge symmetries Lorentz and CPT on the interaction of the fundamental fields conditioned to the restrictions that lead to a renormalizable theory. In the case of Lorentz and CPT symmetries, to the present all experimental indications point to absolute symmetries [3]. However the technological evolution nowadays allows us to test the limit of validity of these particular symmetries to a crescent high degree of precision. Having this in mind the implication of Lorentz and CPT violations have been receiving a lot of attention, specially after the work of Colladay and Kostelecký [4] where a conceptual framework and a procedure for treating spontaneous CPT and Lorentz violations is developed (maintaining gauge structure and renormalizability). Within this framework

a CPT violating extension of the minimal Standard Model is presented and some phenomenological consequences established. In a more recent work Colladay and Kostelecký [5] have continued their theoretical investigations and presented a full Lorentz-violating extension for the Standard Model including CPT-even Lorentz-breaking terms not explicitly presented in their first work. In particular an extended version of the quantum electrodynamics is extracted from the above cited model in such a way that Lorentz and CPT violations are included. As a consequence the usual properties of the fermion and the photon are modified and many measurable implications are considered and discussed. Additional investigations in the subject of breaking the Lorentz invariance have been performed by Colleman and Glashow [6] in a work based on the more general theory of Colladay and Kostelecký [4], [5], but restricted to the special case of rotational and CPT invariance. Tiny Lorentz breaking terms are included in the Lagrangian of the Standard Model in such a way that $SU(3)\otimes SU(2)\otimes U(1)$ gauge invariance, renormalizability by power counting and by anomaly cancellation are maintained. A lack of possible experimental tests of the Lorentz breaking are considered in detail in the high energy regime.

In this very general discussion on the possibilities of breaking Lorentz and CPT invariance one problem received special attention; the possibility of to induce these breaking by radiative corrections in an extended QED, a sector of the extended version of the Standard Model. The Lagrangian of the extended QED, given in the Colladay and Kostelecký [5] investigations, is composed by the usual QED Lagrangian adding three other contributions coming from the breaking possibilities:

$$L^{SB}=-a_\mu\bar{\Psi}\gamma^\mu\Psi-b_\mu\bar{\Psi}\gamma_5\gamma^\mu\Psi+\frac{1}{2}k^\alpha\epsilon_{\alpha\lambda\mu\nu}A^\lambda F^{\mu\nu}$$

where $a_\mu$ and $b_\mu$ are constant (real) prescribed four-vectors, the coupling $k^\alpha$ is real and has dimensions of mass and $\gamma_5$ is the usual Dirac Hermitian matrix related to $\epsilon_{\alpha\lambda\mu\nu}$ by $tr\gamma_5\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu=4i\epsilon_{\alpha\beta\mu\nu}$. An important point about the breaking term of the photon sector in the above Lagrangian is the behavior under potential gauge transformation $(A_\mu\rightarrow A_\mu+\partial_\mu\Lambda)$ [7]. It change by a total derivative leading to an invariant action and to the same equations of motion. This behavior characterize the Chern-Simons form [8]. There are theoretical and experimental aspects related to the phenomenology predicted by the modified theory [9], [4]- [6], [10]. All present analysis seems to point to a zero value for the $k_\mu$ coupling. In searching from Lorentz and CPT effects an immediate question emerges in light of vanishing of the $k_\mu$ in the tree level: is whether the radiative correction coming from another sector of the theory can induce contributions of the Chern-Simons type. This contribution would represent a correction to the photon propagator [5]. From the point of view of the perturbative diagrammatic (Feynman rules) expansion the diagrams corresponding to the modified theory are closely related to the ones of the symmetric theory. They are obtained by insertions on propagators and vertices in the topologies of symmetric theory. The lowest order diagram that represents the correction to the photon propagator is the usual QED vacuum polarization tensor with an insertion or on the internal charged fermion propagator or at the vertices of the diagram. In different words, we need to evaluate the usual one-loop vacuum polarization amplitude but changing the free fermion propagator $S(k)$, obeying the Dirac equation, to a propagator coming from the addition of the $b_\mu$ coupling to the usual QED Lagrangian;

$$G(k)=\frac{i}{\not{k}-m-\not{b}\gamma_5}. \tag{1}$$

Explicitly, we are lead to evaluate the amplitude:

$$\Pi^{\mu\nu}(p)=\int\frac{d^4k}{(2\pi)^4}tr\left\{\gamma^\mu G(k)\gamma^\nu G(k+p)\right\}.$$

The evaluation of this radiative correction has been performed by different authors [5], [7], [11], [12], [13] emphasizing many aspects involved in the calculations. A common conclusion emerges in all of these investigations; the result obtained is not free from ambiguities. The main reason for this is due to the fact that the resulting amplitude $\Pi^{\mu\nu}(p)$ is a divergent object, and in this way, plaged by the very well known problems common to the perturbative calculations in quantum field theories. To evaluate a divergent amplitude the first step we need is to specify a mathematical procedure in which the calculations are turned to be possible. Usually this means to adopt a regularization technique or equivalent philosophy. The choice of such a scheme is guided by the consistency of the procedure in preserving underlying features of the specific theory or general aspects of quantum field theory. There are two dramatic problems involved in the desired consistency: maintenance of symmetry relation and the avoidance of ambiguities related to possible arbitrary choices in the internal momentum routing of the divergent amplitudes. The first aspect is crucial to get a renormalizable theory and second is crucial to establish the power prediction of a quantum field theory, both aspects related to the perturbative solutions. In this context, undoubtedly, the most important technique, specially for the development of gauge quantum field theory, is the Dimensional Regularization [14]. Unfortunately

this is not a general procedure because, among others, the implementation of the $\gamma_5$ matrix is not well defined [15] and, consequently, when this object is present we need to recourse to another procedure. The most famous problem involving this situation is that of triangle anomalies where, when the perturbative evaluation is made in the context of four-dimensional procedure, ambiguities are explicitly isolated [16].

This briefly discussion about these general aspects involved in the evaluation of divergent amplitudes is needed to establish a point of view to our analysis as will became clear in what follows. As have been pointed out by Colladay and Kostelecký [5] the two point diagram $\Pi_{\mu\nu}(p)$, written above with the inclusion of the breaking correction in the propagator, can be viewed in the context of the more fundamental theory from which QED extended has been extracted, as a mathematical structure identical to a one-loop three point diagram coupling to a two photon lines and one axial vector. We can look to the two point diagram as a kinematical limit of the corresponding three point diagram in which the momentum associated to axial-vector leg is zero. With this identification we are lead to evaluate in our calculation mathematical structures identical to two well known amplitudes: the usual QED vacuum polarization tensor and the $AVV$ amplitude related to chiral anomalies and to pion decay phenomenology [7] .

These statement can be put in a very clear way if we write the exact propagators $G(k)$, given in the expression (1), as in the work of Jackiw and Kostelecký [7], in the form

$$G(l) = S(l) + G_b(l)$$

where:

$$G_b(l) = \frac{i}{\not{k} - m - \not{b}\gamma_5} \not{b}\gamma_5 S(l)$$

In this way the amplitude $\Pi_{\mu\nu}(p)$ can be split in three terms

$$\Pi^{\mu\nu} = \Pi_0^{\mu\nu} + \Pi_b^{\mu\nu} + \Pi_{bb}^{\mu\nu} \tag{2}$$

where $\Pi_0^{\mu\nu}$ is the usual QED vacuum polarization tensor and $\Pi_b^{\mu\nu}$ is given by

$$\Pi_b^{\mu\nu}(p) = \int \frac{d^4k}{(2\pi)^4} tr\left\{\gamma^\mu S(l)\gamma^\nu G_b(l+p) + \gamma^\mu G_b(l+p)\gamma^\nu S(l)\right\}$$

where the identification with the mathematical structure of the $AVV$ amplitude, as described above, became now clear. The possible contribution to the induced Chern-Simons term linear in $b_\mu$ ($b_\mu \propto k_\mu$) come from $\Pi_b^{\mu\nu}(p)$. The results of the investigations [5], [6], [7], [11], [12], [13] have been generating two types of controversy: the first it refers to the existence of such CPT and Lorentz violation correction; the other as what is the value of the correction. The first aspect is related to general matters. Colleman and Glashow [5] argued that only a vanishing value of $k_\mu$ is allowed if one demands that the axial current $j_5^\mu(x) = \bar{\Psi}(x)\gamma^\mu\gamma^5\Psi(x)$ should maintain gauge invariance in a Quantum Field Theory for any value of the momenta, or equivalently, at any position $x$. As a consequence of such hypothesis the value of $k_\mu$ should be unambiguous vanishing to first order in $b_\mu$ for any CPT-odd, gauge invariant interaction. On the other hand, Jackiw and Kostelecký [7] argued that a weaker condition could be assumed; in the case when there is no coupling of the axial current with other fields then the physical gauge invariance would be maintained at zero momentum. Put in a different way, the action can be gauge invariant although the Lagrangian density is not. From this point of view $k_\mu$ does not needs to vanish. In this case, what would be its value? This whole question is intimately connected with the fact that, in order to evaluate the correction through perturbative techniques, one needs to deal with divergent integrals. As we have pointed above it is therefore essential, at some point, the definition of a regularization strategy in order to perform the calculation. The final result is clearly subject to the usual problems involved in the treatment of divergent integrals such as ambiguities, for example. For this reason different values for $k_\mu$ have been suggested in the literature. In their paper, Jackiw and Kostelecký [7] explicitly calculate this contribution and find a nonvanishing value. Their calculation is strongly based in the ambiguous character of the mathematical structures involved, when explicitly taking into account surface terms associated to shifts effected in the arbitrary internal momentum routing of the loops. At this point we can say that in all works where the radiatively Chern-Simons term was calculated no unique answer can be given. In a certain way this is not a surprising fact because we are dealing with the typical mathematical indefinitions of the divergences of perturbative solutions of quantum field theories. So, the answer to this question, as well as to any one involving a particular divergent amplitude, cannot be given in a individually way. This means that any regularization scheme or equivalent procedure used to evaluate a particular amplitude needs to be tested in a more general context to prove its consistency. In particular in the QED extended theory the U(1) gauge symmetry is maintained. This implies that the Ward identities associated to vector current conservation (gauge invariance) need to be present in all calculated amplitudes. If this is not the

case the renormalizability of the usual QED would be spoiled by the eventual procedure applied and in consequence the renormalizability of the general theory, the Standard Model, is lost too. Another very important fact related to the mathematical structures involved in the discussed problem, that is crucial for the renormalizability of the full model, is the anomaly cancellation, once we have verified that the linear $b_\mu$ term is closely related to triangular anomaly diagrams. So, we believe that to answer the question related to what is the value for the radiatively induced Chern-Simons term the most general calculation must be given. However only after the consistency of this particular calculation are put in accordance with the more general context a value can be extracted. A negative affirmation is included in this statement: if a procedure do not furnishes a consistent result for a particular amplitude of the theory then the results produced by this procedure to any other amplitude cannot be taken seriously for any purposes.

The purpose of this contribution is to investigate in a more general context of perturbative calculations what is the possible role played by the ambiguities when the general aspects related to renormalizability are considered. To evaluate the amplitude we will use a strategy to calculate and manipulate divergent integrals in which no divergent integrals are, in fact, calculated. In the result produced by our technique it will be possible to map the results produced by others authors using different schemes.

The specific implication of our argument to the discussed problem is that we need to evaluate the term $\Pi_0^{\mu\nu}$ and $\Pi_b^{\mu\nu}$ in the expression (2) in a way that the results produced in both cases are required to be consistent simultaneously. Our main guide in this investigation is the maintenance of the U(1) gauge symmetry in the usual QED theory. In order to define a consistent strategy to handle the divergent amplitudes we first treat the vacuum polarization tensor $\Pi_0^{\mu\nu}$ in such a way that any specific assumption about a regulator is avoided. In intermediary steps we only make use of very general properties of a such eventual regulator. The divergent content of the amplitudes is separated from the finite one by the use of a identity at the level of the integrand. In the final form so obtained, no dependence with external momenta are present in a set of basic divergent objects that we will define. In respect to finite terms, they are in this way, free from effects of regularization. In our final results those of Jackiw and Kostelecký [7] will be recovered in detail. After all calculations have been performed we show how the choice of demanding of gauge invariance for the usual vacuum polarization tensor will lead us to a nonzero value ambiguity free for the Chern-Simons term, but if in addition we choose to require consistency with the anomaly cancellation a vanishing value for the Chern-Simons radiatively induced contribution can be obtained.

## II. THE QED VACUUM POLARIZATION TENSOR WITH ARBITRARY INTERNAL MOMENTUM ROUTING

Let us now evaluate the well known Q.E.D. vacuum polarization tensor to one loop order from the point of view of our proposed strategy [17]. We perform the calculations allowing complete arbitrariness in the internal momentum routing. Explicitly we have

$$\left(\Pi_0\right)_{\mu\nu} = \int \frac{d^4k}{\left(2\pi\right)^4} tr\left\{\gamma_\mu S(k+k_1)\gamma_\nu S(k+k_2)\right\}, \tag{3}$$

where $S(k)$ is a usual 1/2 spin free fermion propagator, carrying momentum $k$. Of course if an ambiguity free result is required only the combination $(k_1-k_2)$ should be allowed. Next, we can put $\Pi_0^{\mu\nu}$ in a more convenient form for our purposes. Explicitly:

$$\begin{aligned}
\left(\Pi_0\right)_{\mu\nu} = 4\Bigg\{ & \int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{2k_\mu k_\nu}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]} \\
& +(k_1+k_2)_\nu \int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]} \\
& +(k_2+k_1)_\mu \int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\nu}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]} \\
& +\left(k_{2\mu}k_{1\nu}+k_{1\mu}k_{2\nu}\right)\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]}\Bigg\} \\
& -8g_{\mu\nu}\Bigg\{\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+k_1)^2-m^2]}+\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+k_2)^2-m^2]} \\
& -(k_1-k_2)^2\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]}\Bigg\}.
\end{aligned} \tag{4}$$

To obtain the above expression we have evaluated the traces over the Dirac matrices and used the following identity:

$$(k+k_i)\cdot(k+k_j)=\frac{1}{2}\left[(k+k_i)^2-m^2\right]+\frac{1}{2}\left[(k+k_j)^2-m^2\right]-\frac{1}{2}\left[(k_i-k_j)^2-2m^2\right]. \tag{5}$$

In the expression (4) we can identify a set of divergent integrals with degree of divergence that includes linear and quadratic ones. In this way, special attention needs to be given to the question of ambiguities related to possible shifts in the loop momentum. At this point the usual procedure is to adopt some regularization philosophy. To avoid specific choices from now on we will consider all divergent integrals to be regulated in 4-D by some implicit function, such that

$$\int\frac{d^4k}{(2\pi)^4}f(k)\to\int\frac{d^4k}{(2\pi)^4}f(k)\left\{\lim_{\Lambda_i^2\to\infty}G_{\Lambda_i}\left(k^2,\Lambda_i^2\right)\right\}=\int_\Lambda\frac{d^4k}{(2\pi)^4}f(k). \tag{6}$$

Here $\Lambda_i's$ are parameters of a distribution $G(k^2,\Lambda_i^2)$ whose asymptotic behavior in $k$ renders the integral finite. Besides this, we require the existence of a connection limit

$$\lim_{\Lambda_i^2\to\infty}G_{\Lambda_i}\left(k^2,\Lambda_i^2\right)=1. \tag{7}$$

This condition, in particular, guarantees that the finite integrals value will not be modified. We can make use of this property to extract the part of the amplitude that carries the external momentum dependence without any contamination by regularization. Having this in mind, after adopting the implicit presence of a generic distribution, we treat all integrals by using identities at level of the integrand in such a way as to obtain a mathematical structure where the dependence on external momenta are contained only in finite integrals [1]. In its most general form this identity reads

$$\frac{1}{\left[(k+k_i)^2-m^2\right]}=\frac{1}{(k^2-m^2)}+\sum_{j=1}^{N}\frac{(-1)^j\left(k_i^2+2k_i\cdot k\right)^j}{(k^2-m^2)^{j+1}}+\frac{(-1)^{N+1}\left(k_i^2+2k_i\cdot k\right)^{N+1}}{(k^2-m^2)^{N+1}\left[(k+k_i)^2-m^2\right]}, \tag{8}$$

where $k_i$ is an arbitrary momentum. The value of $N$ should be chosen so that the last term in the above expression now corresponds to a finite integral, and then, by virtue of (7), the integration can be performed without restriction (the integrals so handled can be found in appendix B). With respect to divergent parts, no additional supposition is made and all terms are written as combinations of five basic objects [17], [19]:

$$\begin{aligned}\bullet\ \Box_{\alpha\beta\mu\nu}=&\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{24k_\mu k_\nu k_\alpha k_\beta}{(k^2-m^2)^4}-g_{\alpha\beta}\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{4k_\mu k_\nu}{(k^2-m^2)^3}\\&-g_{\alpha\nu}\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{4k_\beta k_\mu}{(k^2-m^2)^3}-g_{\alpha\mu}\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{4k_\beta k_\nu}{(k^2-m^2)^3}\end{aligned} \tag{9}$$

$$\bullet\Delta_{\mu\nu}=\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{4k_\mu k_\nu}{(k^2-m^2)^3}-\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}}{(k^2-m^2)^2} \tag{10}$$

$$\bullet\nabla_{\mu\nu}=\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{2k_\nu k_\mu}{(k^2-m^2)^2}-\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}}{(k^2-m^2)} \tag{11}$$

$$\bullet\ I_{log}(m^2)=\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)^2} \tag{12}$$

$$\bullet I_{quad}(m^2)=\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)}. \tag{13}$$

This way it is possible to make contact with the corresponding results with other traditional techniques if the above objects are evaluated in the adopted scheme. The main advantage of this strategy is to manipulate each divergent

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[1]The philosophy is somewhat close in spirit to the BPHZ procedure [18] where a Taylor expansion is made around the value of the physical momentum $p=0$.

integral without explicit calculations of a divergent integral. This decomposition is sufficient for the required analysis, and the same expression for a given integral is used in all places where it occurs. The price we pay is that the finite integrals at first sight seem to be more complicated although they can be readily organized and calculated using standard techniques. After this essential discussion we turn our attention back to the calculation of $\Pi_0^{\mu\nu}$. Using the results for the integrals obtained according to this procedure (appendix B) we can write:

$$
\begin{aligned}
(\Pi_0)_{\mu\nu} &= \frac{4}{3}[(k_1-k_2)^2 g_{\mu\nu} - (k_1-k_2)_\mu (k_1-k_2)_\nu] \times \\
&\times \left\{ [I_{log}(m^2)] - \left(\frac{i}{(4\pi)^2}\right)\left[\frac{1}{3} + \frac{(2m^2+(k_1-k_2)^2)}{(k_1-k_2)^2} Z_0(m^2,m^2,(k_1-k_2)^2)\right]\right\} \\
&+ A_{\mu\nu},
\end{aligned}
\tag{14}
$$

where

$$
\begin{aligned}
A_{\mu\nu} = +4[\nabla_{\mu\nu}] &+ (k_1-k_2)_\alpha (k_1-k_2)_\beta \left[\frac{1}{3}\Box_{\alpha\beta\mu\nu} + \frac{1}{3}\triangle_{\mu\beta} g_{\alpha\nu} + g_{\alpha\mu}\triangle_{\beta\nu} - g_{\mu\nu}\triangle_{\alpha\beta} - \frac{2}{3} g_{\alpha\beta}\triangle_{\mu\nu}\right] \\
&+ [(k_1-k_2)_\alpha (k_1+k_2)_\beta - (k_1+k_2)_\alpha (k_1-k_2)_\beta]\left[\frac{1}{3}\Box_{\alpha\beta\mu\nu} + \frac{1}{3}\triangle_{\mu\beta} g_{\nu\alpha} + \frac{1}{3}\triangle_{\beta\nu} g_{\alpha\mu}\right] \\
&+ (k_1+k_2)_\alpha (k_1+k_2)_\beta \left[\Box_{\alpha\beta\mu\nu} - \triangle_{\nu\alpha} g_{\mu\beta} - \triangle_{\beta\nu} g_{\alpha\mu} - 3\triangle_{\alpha\beta} g_{\mu\nu}\right].
\end{aligned}
\tag{15}
$$

The definition of the finite function $Z_k$ can be found in appendix A. In the above result it becomes clear that the presence of the objects $\Box, \nabla$ and $\Delta$, which are differences between divergent integrals with the same degree of divergence, renders the Q.E.D. vacuum polarization tensor ambiguous and break Ward identities $(k_1-k_2)^\mu (\Pi_0)_{\mu\nu} = (k_1-k_2)^\nu (\Pi_0)_{\mu\nu} = 0$, which represent violation of U(1) gauge invariance. It is important to note that there are three different kinds of violating terms. One of them involves an ambiguous combination of the momenta $k_1$ and $k_2$. Another one where only their differences appear (external momenta) and a third where no dependence on such momenta are present. In view of this, it is easy to check that it is not possible to restore gauge invariance by choosing values for $k_1$ and $k_2$ (arbitrary) momenta. The ONLY possibility we have is the requirement that the value assumed by the objects $\Box, \nabla$ and $\Delta$ should be simultaneously zero. After assuming these "consistency conditions" the remaining result for $\Pi_0^{\mu\nu}$ can be immediately identified with the corresponding one produced by Dimensional Regularization (D.R.) technique (after expressing $I_{log}(m^2)$ in this specific mathematical language). This natural mapping is a consequence of the fact that our three differences $\Box, \nabla$ and $\Delta$ vanish in D.R. For a more detailed discussion on the ambiguities and symmetry violations in this framework we report the reader to references [17], [19], [21].

We can summarize what we have learned in our analysis of this particular amplitude (and many others performed elsewhere) by the sentence: There is no chance of consistency in a perturbative calculation without requiring what we call the consistency conditions. Ambiguities associated with arbitrary choices of internal momenta and gauge invariance constitutes only two aspects that can be associated to them [17], [19]. Having this in mind in what follows we calculate the second term in the expression (2), using the same prescriptions above.

## III. EXPLICIT CALCULATION OF THE AXIAL-VECTOR-VECTOR TRIANGLE

The first order contribution to the Chern-Simons term comes from the $\Pi_b^{\mu\nu}$ amplitude, the second term in expression (2). It is explicitly given by [7]

$$
\begin{aligned}
\Pi_b^{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} tr \{\gamma^\mu S(k)\gamma^\nu G_b(k+p) \\
+ \gamma^\mu G_b(k)\gamma^\nu S(k+p)\},
\end{aligned}
\tag{16}
$$

where

$$
G_b(k) = \frac{1}{\not k - m - \not b\gamma_5}\ \not b\gamma_5 S(k). \tag{17}
$$

To evaluate $\Pi_b^{\mu\nu}$ to lowest order in $b$, we simply replace the above expression by

$$
G_b(k) = -iS(k)\ \not b\gamma_5 S(k). \tag{18}
$$

Now, the corresponding expression to $\Pi_b^{\mu\nu}$ may be written as:

$$\Pi_b^{\mu\nu}(p) \simeq b_\lambda \Pi^{\mu\nu\lambda}(p), \tag{19}$$

where

$$\Pi^{\mu\nu\lambda}(p) = (-i) \int \frac{d^4k}{(2\pi)^4} tr \left\{ \gamma^\mu S(k) \gamma^\nu S(k+p) \gamma^\lambda \gamma_5 S(k+p) + \right. \\ \left. + \gamma^\mu S(k) \gamma^\lambda \gamma_5 S(k) \gamma^\nu S(k+p) \right\}. \tag{20}$$

These two terms can be identified as a particular kinematic situations of the Axial-Vector-Vector triangular amplitude [5], [7], related by (the anomalous) Ward identity to pion decay phenomenology [17]. In virtue of this we will consider the most general calculation of the $AVV$ amplitude. Only at the end of the calculation we will return to the above specific situations. We believe that proceeding this way our conclusions may be more general and transparent. We start by giving the definition:

$$T_{\lambda\mu\nu}^{AVV} = -\int \frac{d^4k}{(2\pi)^4} tr \left\{ \gamma_\mu \left[ (\not k + \not k_1) - m \right]^{-1} \gamma_\nu \left[ (\not k + \not k_2) - m \right]^{-1} \gamma_\lambda \gamma_5 \left[ (\not k + \not k_3) - m \right]^{-1} \right\}, \tag{21}$$

where $k_1, k_2$ and $k_3$ stand for arbitrary choices of internal lines momenta. They are related to external ones by their differences. From now on we will follow strictly the same steps used in the preceding section. After taking the Dirac traces and using the identity (5) we organize the expression in the following and convenient form

$$T_{\lambda\mu\nu}^{AVV} = -4i \left\{ -F_{\lambda\mu\nu} + N_{\lambda\mu\nu} + M_{\lambda\mu\nu} + P_{\lambda\mu\nu} \right\}, \tag{22}$$

where we have introduced the definitions

$$\bullet\, P_{\lambda\mu\nu} = g_{\mu\nu} \varepsilon_{\alpha\beta\lambda\xi} \int_\Lambda \frac{d^4k}{(2\pi)^4} \frac{(k+k_1)_\alpha (k+k_2)_\beta (k+k_3)_\xi}{[(k+k_1)^2 - m^2][(k+k_2)^2 - m^2][(k+k_3)^2 - m^2]} \tag{23}$$

$$\bullet F_{\lambda\mu\nu} = \int_\Lambda \frac{d^4k}{(2\pi)^4} \{ \varepsilon_{\nu\beta\lambda\xi} (k+k_1)_\mu (k+k_2)_\beta (k+k_3)_\xi \\ + \varepsilon_{\mu\beta\lambda\xi} (k+k_1)_\nu (k+k_2)_\beta (k+k_3)_\xi \\ + \varepsilon_{\mu\alpha\nu\beta} (k+k_1)_\alpha (k+k_2)_\beta (k+k_3)_\lambda \\ \left. + \varepsilon_{\mu\alpha\nu\xi} (k+k_1)_\alpha (k+k_3)_\xi (k+k_2)_\lambda \right\} \times \\ \times \left\{ \frac{1}{\left[ (k+k_1)^2 - m^2 \right] \left[ (k+k_2)^2 - m^2 \right] \left[ (k+k_3)^2 - m^2 \right]} \right\} \tag{24}$$

$$\bullet N_{\lambda\mu\nu} = \frac{\varepsilon_{\mu\alpha\nu\lambda}}{2} \left\{ \int_\Lambda \frac{d^4k}{(2\pi)^4} \frac{(k+k_1)_\alpha}{[(k+k_2)^2 - m^2][(k+k_1)^2 - m^2]} \right. \\ + \int_\Lambda \frac{d^4k}{(2\pi)^4} \frac{(k+k_1)_\alpha}{[(k+k_1)^2 - m^2][(k+k_3)^2 - m^2]} \\ + [2m^2 - (k_2 - k_3)^2] \times \\ \left. \times \int \frac{d^4k}{(2\pi)^4} \frac{(k+k_1)_\alpha}{[(k+k_1)^2 - m^2][(k+k_2)^2 - m^2][(k+k_3)^2 - m^2]} \right\} \tag{25}$$

$$\bullet M_{\lambda\mu\nu} = m^2 \varepsilon_{\mu\nu\alpha\lambda} \int \frac{d^4k}{(2\pi)^4} \frac{\{ (k+k_2)_\alpha - (k+k_1)_\alpha + (k+k_3)_\alpha \}}{\left[ (k+k_1)^2 - m^2 \right] \left[ (k+k_2)^2 - m^2 \right] \left[ (k+k_3)^2 - m^2 \right]}. \tag{26}$$

Looking at the above expression we can identify a set of Feynman integrals some of them are divergent. The most severe degree of divergence in this case is the linear one, which is contained in $N_{\lambda\mu\nu}$, in the two point function structures. We use here the same results adopted to treat the $\Pi_0^{\mu\nu}$ amplitude. After the evaluation of those integrals corresponding to three point function structures (appendix B) we arrive at the followings forms:

$$\bullet\, P_{\lambda\mu\nu} = 0 \tag{27}$$

$$\bullet M_{\lambda\mu\nu} = -\left(\frac{i}{(4\pi)^2}\right)\varepsilon_{\mu\alpha\nu\lambda} m^2 \left\{(k_2-k_1)_\alpha\left(\xi_{00}-\xi_{01}\right) + (k_3-k_1)_\alpha\left(\xi_{00}-\xi_{10}\right)\right\} \tag{28}$$

$$\begin{aligned}\bullet N_{\lambda\mu\nu} = {} & \frac{\varepsilon_{\mu\alpha\nu\lambda}}{4}(k_1-k_2)_\alpha\left\{I_{\log}\left(m^2\right) - \left(\frac{i}{(4\pi)^2}\right) Z_0\left((k_1-k_2)^2;m^2\right)\right. \\ & \left. + \left(\frac{i}{(4\pi)^2}\right)\left[2m^2-(k_3-k_2)^2\right](2\xi_{01})\right\} \\ & -\frac{\varepsilon_{\mu\alpha\nu\lambda}}{4}(k_3-k_1)_\alpha\left\{I_{\log}\left(m^2\right) - \left(\frac{i}{(4\pi)^2}\right) Z_0\left((k_1-k_3)^2;m^2\right)\right. \\ & \left. + \left(\frac{i}{(4\pi)^2}\right)\left[2m^2-(k_3-k_2)^2\right](2\xi_{10})\right\} \\ & -\frac{\varepsilon_{\mu\alpha\nu\lambda}}{4}\left[(k_1+k_2)_\beta + (k_3+k_1)_\beta\right]\Delta_{\alpha\beta}\end{aligned} \tag{29}$$

$$\begin{aligned}\bullet F_{\lambda\mu\nu} = {} & \left(\frac{i}{(4\pi)^2}\right)(k_3-k_1)_\xi (k_2-k_1)_\beta \left\{\varepsilon_{\nu\beta\lambda\xi}[(k_2-k_1)_\mu\left(\xi_{02}+\xi_{11}-\xi_{01}\right)\right. \\ & + (k_3-k_1)_\mu\left(\xi_{20}+\xi_{11}-\xi_{10}\right)] \\ & +\varepsilon_{\mu\beta\lambda\xi}[(k_2-k_1)_\nu\left(\xi_{02}+\xi_{11}-\xi_{01}\right) \\ & + (k_3-k_1)_\nu\left(\xi_{20}+\xi_{11}-\xi_{10}\right)] \\ & +\varepsilon_{\mu\beta\nu\xi}[(k_3-k_1)_\lambda\left(\xi_{11}-\xi_{20}+\xi_{10}\right) \\ & \left. + (k_2-k_1)_\lambda\left(\xi_{02}-\xi_{11}-\xi_{01}\right)]\right\} \\ & -\frac{\varepsilon_{\mu\nu\lambda\xi}}{4}\left\{\left((k_3-k_1)_\xi + (k_2-k_1)_\xi\right)\varphi_0\right\} \\ & +\varepsilon_{\nu\beta\lambda\sigma}(k_2-k_3)_\beta\frac{\Delta_{\mu\sigma}}{4} + \varepsilon_{\mu\beta\lambda\sigma}(k_2-k_3)_\beta\frac{\Delta_{\nu\sigma}}{4} + \\ & +\varepsilon_{\mu\sigma\nu\beta}\left[(k_2-k_1)_\beta + (k_3-k_1)_\beta\right]\frac{\Delta_{\lambda\sigma}}{4}\end{aligned} \tag{30}$$

with

$$\varphi_0 = I_{\log}\left(m^2\right) + \left(\frac{i}{(4\pi)^2}\right)\left\{-Z_0\left((k_2-k_3)^2;m^2\right) + 2\left[\frac{1}{2}+m^2\xi_{00}\right] - (k_3-k_1)^2\xi_{10} - (k_2-k_1)^2\xi_{01}\right\}. \tag{31}$$

The functions $\xi_{mn}$ are related to the finite content of three point functions and are defined in appendix A. The complete solution to $AVV$ can be written in the following form

$$\begin{aligned}\frac{T^{AVV}_{\lambda\mu\nu}}{-4i} = {} & \left(\frac{i}{(4\pi)^2}\right)(k_3-k_1)_\xi (k_2-k_1)_\beta \left\{\varepsilon_{\nu\lambda\beta\xi}[(k_3-k_1)_\mu\left(\xi_{20}+\xi_{11}-\xi_{10}\right)\right. \\ & + (k_2-k_1)_\mu\left(\xi_{11}+\xi_{02}-\xi_{01}\right)] \\ & +\varepsilon_{\mu\lambda\beta\xi}[(k_3-k_1)_\nu\left(\xi_{11}+\xi_{20}-\xi_{10}\right) \\ & + (k_2-k_1)_\nu\left(\xi_{02}+\xi_{11}-\xi_{01}\right)] \\ & +\varepsilon_{\mu\nu\beta\xi}[(k_3-k_1)_\lambda\left(\xi_{11}-\xi_{20}+\xi_{10}\right) \\ & \left. + (k_2-k_1)_\lambda\left(\xi_{02}-\xi_{01}-\xi_{11}\right)]\right\} \\ & + \left(\frac{i}{(4\pi)^2}\right)\frac{\varepsilon_{\mu\nu\lambda\beta}}{4}(k_3-k_1)_\beta\left\{Z_0\left((k_1-k_3)^2;m^2\right) - Z_0\left((k_2-k_3)^2;m^2\right)\right. \\ & + \left[2(k_3-k_2)^2-(k_1-k_3)^2\right]\xi_{10} + \\ & \left. + (k_1-k_2)^2\xi_{01} + \left[1-2m^2\xi_{00}\right]\right\}\end{aligned} \tag{32}$$

$$
\begin{aligned}
&+\left(\frac{i}{(4\pi)^2}\right)\frac{\varepsilon_{\mu\nu\lambda\beta}}{4}(k_2-k_1)_\beta\Big\{Z_0\left((k_1-k_2)^2;m^2\right)-Z_0\left((k_2-k_3)^2;m^2\right)\\
&\qquad\qquad\qquad\qquad\qquad\qquad+\left[2(k_3-k_2)^2-(k_1-k_2)^2\right]\xi_{01}\\
&\qquad\qquad\qquad\qquad\qquad\qquad+\,(k_3-k_1)^2\xi_{10}+\left[1-2m^2\xi_{00}\right]\Big\}\\
&-\frac{\varepsilon_{\mu\nu\beta\sigma}}{4}\left[(k_2-k_1)_\beta+(k_3-k_1)_\beta\right]\Delta_{\lambda\sigma}\\
&+\frac{\varepsilon_{\nu\lambda\beta\sigma}}{4}(k_2-k_3)_\beta\Delta_{\mu\sigma}+\frac{\varepsilon_{\mu\lambda\beta\sigma}}{4}(k_2-k_3)_\beta\Delta_{\nu\sigma}\\
&-\frac{\varepsilon_{\mu\nu\lambda\alpha}}{4}\left[(k_1+k_2)_\beta+(k_3+k_1)_\beta\right]\Delta_{\alpha\beta}.
\end{aligned}
$$

It is important to emphasize that *this result is the most general one for the $AVV$ amplitude. All possible choices for specific routing or shifts on loop momentum are automatically included. In addition this result can be mapped on those produced by specific regularization once the divergent objects are maintained intact.* They are contained in the object $\Delta_{\alpha\beta}$.

In this result it is important to note the presence of a potentially ambiguous term, the last one in the above expression. This is due to the fact that the combinations of the $k_1$, $k_2$ and $k_3$ involved are not restricted to differences between them. As in the previously analyzed $\Pi_0^{\mu\nu}$ amplitude this term is a coefficient of the piece $\Delta$. In addition we can note that there is another term in the expression where the object $\Delta$ is present but with non ambiguous coefficients. Before more detailed analysis let us now to proceed in order to obtain the corresponding result for the Chern-Simons contribution. As an initial step in this direction we take the choices: $k_1=0$, $k_2=k_3=p$ in order to reproduce the kinematical situation corresponding to the first contribution for $\Pi^{\mu\nu\lambda}(p)$ in eq.(20). After this, in the obtained expression for $T_{\mu\nu\lambda}^{AVV}$, we make $k_1=p$, $k_2=k_3=0$ and interchange $\mu\leftrightarrow\nu$ to obtain the second contribution to eq.(20). Adding the results so obtained we get

$$
\begin{aligned}
\Pi_{\lambda\mu\nu}(p)&=2i\left\{\varepsilon_{\mu\nu\beta\sigma}p^\beta\Delta_{\lambda\sigma}-\varepsilon_{\mu\nu\lambda\beta}p^\sigma\Delta_{\beta\sigma}\right\}\\
&\quad+\left(\frac{1}{4\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p^\beta\left\{Z_0\left(p^2;m^2\right)-p^2\left(\xi_{01}+\xi_{10}\right)+\left[1-2m^2\left(\xi_{00}\right)\right]\right\}.
\end{aligned}
\tag{33}
$$

The contribution to the Chern-Simons term is extracted from the above expression taking $p^2=0$. Using the properties $Z_0(p^2=0)=0$ and $\xi_{00}(p^2=0)=\frac{-1}{2m^2}$, the result is:

$$
\Pi_{\lambda\mu\nu}(p)=\left(\frac{1}{2\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p^\beta+2i\left\{\varepsilon_{\mu\nu\beta\sigma}p^\beta\Delta_{\lambda\sigma}-\varepsilon_{\mu\nu\lambda\beta}p^\sigma\Delta_{\beta\sigma}\right\}
\tag{34}
$$

As previously announced, from our result its easy to obtain the corresponding result of Jackiw and Kostelecký. This result emerges immediately if we identify $-\Delta_{\xi\beta}$ with the surface term evaluated explicitly in ref. [7]

$$
\begin{aligned}
\Delta S_{\mu\nu}&=\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{\partial}{\partial k_\nu}\left(\frac{k_\mu}{(k^2-m^2)^2}\right)=\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{-4k_\mu k_\nu}{(k^2-m^2)^3}+\int_\Lambda\frac{d^4k}{(2\pi)^4}\frac{g_{\mu\nu}}{(k^2-m^2)^2}\\
&=\left(\frac{i}{(4\pi)^2}\right)\left(\frac{1}{2}\right)g_{\mu\nu}
\end{aligned}
\tag{35}
$$

which leads us to

$$
\Pi_{\mu\nu\lambda}(p)=\left(\frac{3}{8\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p^\beta.
\tag{36}
$$

As should require to demonstrate the correctness of our approach.

If we stop the investigation at this point our calculations would represent one point of view for the problem identical to that adopted by Jackiw and Kostelecký. Our contribution for this discussion would be reduced to just a more explicitly evaluation of the involved mathematical structures. It is time to add some new considerations for the discussions to deserve some attention of the reader.

Let us then turn the attention back to our result but now following our arguments. The first aspect is about the value of $\Delta_{\xi\beta}$. Our discussion in the previous section taught us that a nonvanishing value for $\Delta_{\xi\beta}$ will not only render

the Q.E.D. vacuum polarization tensor ambiguous but also will necessarily violate its gauge invariance. Therefore in order to maintain U(1) gauge invariance of $\Pi_0^{\mu\nu}$ we have to choose $\Delta_{\mu\nu} = 0$ (which, as we have said, is immediately obtained in D.R.). Consequently the value for the Chern-Simons term, eq.(34), is given by

$$\Pi_{\mu\nu\lambda}(p) = \left(\frac{1}{2\pi^2}\right) \varepsilon_{\mu\nu\lambda\beta} p^\beta. \tag{37}$$

The above results constitutes now, in fact, an alternative value for the Chern-Simons term once it is different from those presented by other authors in the literature. It is important to stress that to arrive at this value we have only chosen that the calculational method used to evaluate the divergent amplitudes exhibits consistency in a larger context of the perturbative calculations. We can say that the so obtained result is dictated by the consistency requirement of the pure QED. In other words, the properties adopted to purely divergent integrals are necessary conditions to give meaningless to QED in the perturbative approach. Any regularization scheme, or equivalent philosophy that do not furnishes $\Box = \nabla = \Delta = 0$ cannot be accepted for the treatment of the QED (in particular) divergences. In the D.R. technique these relations are automatically incorporated and this is exactly the reason for the success of this method in what concern to the ambiguity elimination and symmetry relations (Ward Identities) maintenance. In other QED consistent techniques like Pauli-Villars [21] recipe, in the last instance, all we require to the coefficients of the superposition that characterizes the method, is that $\Box = \nabla = \Delta = 0$, which can be easily verified [21]. The relevant question, in light of these statements, is: can we accept a result produced by a strategy to handle divergences in perturbative calculations of QFT, for a particular problem, if the adopted strategy cannot lead us to a QED consistent results? If the answer is no, then the value in the eq.(37) represents a nonzero value for the Chern-Simons term free from the ambiguities related to arbitrary choices of the internal lines momenta in the loops. There is a very attractive aspect involved once we need not to admit the lost of the translational invariance in perturbative calculations, the most basic one implemented in a QFT. In spite of the preceding argumentation is bounded to QED, which is very strong by itself, we believe that these aspects are deeply and directly related to our problem for many reasons. Firstly $\Pi_0^{\mu\nu}$ present in the full amplitude, eq.(2), is a QED amplitude requiring then an identical treatment. For the second we can call to the discussions an important memory reason, the extended Standard Model, which have generate the discussion about the Chern-Simons term radiatively induced, developed by Colladay and Kostelecký, were constructed in light of the renormalizability and, therefore, all assumptions that spoils the QED renormalizability destroys also the fundamental basis of the discussion. At this point if we take seriously the last comment then this is not the whole history. In the ref. [5] it was conjectured by Colladay and Kostelecký that the imposition of the anomaly cancellation consistency requirement, an important ingredient of the Standard Model, should lead to a vanishing value for the Chern-Simons term. Once the result present in our calculations, eq.(32), for the mathematical structures involved is the most general one, we are ready to investigate also this aspect.

Up to now we have not asked what the U(1) gauge invariance of the $AVV$ amplitude can reveal to us? In different words; U(1) gauge invariance was assumed as an essential ingredient and is crucial in connection with the renormalizability by anomaly cancellation in the theory that has generated the amplitude $\Pi^{\mu\nu}$. Then it seems not to make sense to attribute any significance to the result of a particular calculation that lost the initial ingredient, used as a guide to try to find a candidate to Lorentz invariance breaking phenomena. In summary: does the $AVV$ amplitude remain U(1) gauge invariant after the calculations? To adequately answer we need to verify its corresponding Ward identities. This is therefore what we will consider in the next section.

## IV. WARD IDENTITIES AND ANOMALIES

In order to answer our present question we have to explicitly verify the Ward identities associated with $AVV$. This is obtained by contracting the expression (32) with the corresponding external momenta where from now on we adopt the definitions $k_3 - k_1 = q$, $k_1 - k_2 = p$, $k_3 - k_2 = p + q$ once only physical momenta remains in the expression after taking $\Delta = 0$. The detailed explicit evaluation of such identities is straightforward [17], [20] although some algebraic effort is involved. The basic ingredients for this calculation involves properties of the functions $\xi_{mn}$ and $Z_k$. They are

$$\bullet\, q^2\left(\xi_{11}\right) - (p\cdot q)\left(\xi_{02}\right) = \frac{1}{2}\left\{\frac{-1}{2} Z_0\left((p+q)^2; m^2\right) + \frac{1}{2} Z_0\left(p^2; m^2\right) + q^2\left(\xi_{01}\right)\right\} \tag{38}$$

$$\bullet\, q^2\left(\xi_{20}\right) - (p\cdot q)\left(\xi_{11}\right) = \frac{1}{2}\left\{-\left[\frac{1}{2} + m^2 \xi_{00}\right] + \frac{p^2}{2}\left(\xi_{01}\right) + \frac{3q^2}{2}\left(\xi_{10}\right)\right\} \tag{39}$$

$$\bullet\, p^2\left(\xi_{02}\right) - (p\cdot q)\left(\xi_{11}\right) = \frac{1}{2}\left\{-\left[\frac{1}{2} + m^2 \xi_{00}\right] + \frac{q^2}{2}\left(\xi_{10}\right) + \frac{3p^2}{2}\left(\xi_{01}\right)\right\} \tag{40}$$

$$\bullet p^2\left(\xi_{11}\right)-(p\cdot q)\left(\xi_{20}\right) = \frac{1}{2}\left\{-\frac{1}{2}Z_0\left((p+q)^2;m^2\right)+\frac{1}{2}Z_0\left(q^2;m^2\right)+p^2\xi_{10}\right\} \tag{41}$$

$$\bullet q^2\left(\xi_{10}\right)-(p\cdot q)\left(\xi_{01}\right) = \frac{1}{2}\left\{-Z_0\left((p+q)^2;m^2\right)+Z_0\left(p^2;m^2\right)+q^2\left(\xi_{00}\right)\right\} \tag{42}$$

$$\bullet p^2\left(\xi_{01}\right)-(p\cdot q)\left(\xi_{10}\right) = \frac{1}{2}\left\{-Z_0\left((p+q)^2;m^2\right)+Z_0\left(q^2;m^2\right)+p^2\left(\xi_{00}\right)\right\}. \tag{43}$$

The result so obtained is:

$$\bullet\, p^\nu T^{AVV}_{\lambda\mu\nu} = -\left(\frac{1}{8\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p^\beta q^\nu \tag{44}$$

$$\bullet q^\mu T^{AVV}_{\lambda\mu\nu} = -\left(\frac{1}{8\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p^\beta q^\mu \tag{45}$$

$$\bullet\,(p+q)^\lambda T^{AVV}_{\lambda\mu\nu} = -2m\left\{T^{PVV}_{\mu\nu}\right\}, \tag{46}$$

where

$$T^{PVV}_{\mu\nu} = \left(\frac{-1}{4\pi^2}\right)m\varepsilon_{\mu\nu\lambda\beta}p^\beta q^\lambda\xi_{00}, \tag{47}$$

so the axial vector identity is satisfied and the two gauge identities violated.

These expressions do not constitute a surprising fact, once its the well known phenomena of anomaly involved in the pion decay [16]. We only call attention to the way that they are obtained; completely off the mass shell, without the use of explicit regularizations and using a procedure that treats all amplitudes in all theories and models according to the same point of view.

The main aspect involved in our present discussion is the fact that we have lost the U(1) gauge invariance of the amplitude in the calculation. To attribute physical significance, in connection to the pion decay phenomenology or Sutherland-Veltman paradox, a redefinition needs to be given by the inclusion of an anomalous term that allows us to recover the vector Ward identities [16] corresponding to our required U(1) gauge invariance. This could be achieved by the substitution

$$\left(T^{AVV}_{\lambda\mu\nu}(p,q)\right)_{phys} = T^{AVV}_{\lambda\mu\nu}(p,q)-T^{AVV}_{\lambda\mu\nu}(0). \tag{48}$$

Here $\left(T^{AVV}_{\lambda\mu\nu}\right)_{phys}$ is a redefinition of the amplitude and $T^{\lambda\mu\nu}_{AVV}(0)$ is its value at $p^2=q^2=0$, namely

$$\left(T^{AVV}_{\lambda\mu\nu}\right)(0) = \left(\frac{1}{8\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}\left[q^\beta-p^\beta\right]. \tag{49}$$

After these considerations, the Ward identities will become:

$$\bullet\, p^\nu\left(T^{AVV}_{\lambda\mu\nu}\right)_{phy} = 0 \tag{50}$$

$$\bullet q^\mu\left(T^{AVV}_{\lambda\mu\nu}\right)_{phy} = 0 \tag{51}$$

$$\bullet\,(p+q)^\lambda\left(T^{AVV}_{\lambda\mu\nu}\right)_{phy} = -2m\left\{T^{PVV}_{\mu\nu}\right\}-\left(\frac{1}{4\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}\left[p^\lambda q^\beta\right]. \tag{52}$$

The modified $AVV$ amplitude by the inclusion of the anomalous term is now in agreement with the phenomenology, so the right hand side of eq.(52), predicts the pion decay. This modified $AVV$ amplitude is used to construct the renormalizable standard model by anomaly cancellation.

After all these calculations a question is in order: in what sense $AVV$ triangle anomaly affect the Chern-Simons term? From the physical point of view a QFT in the perturbative approach is in last instance, a collection of basic amplitudes. These amplitudes are nothing more than mathematical structures (Green's functions) connected to the phenomenology by the insertion of the external lines which characterizes the physical processes. In a theory like the Standard Model different physical processes may be described perturbatively making use of the same intermediate Green's functions. The value attributed to the basic mathematical structures cannot be associated to the particular physical processes involved but needs to have the same value in all places of occurrence. The amplitude $\Pi^{\mu\nu}_b$, responsible for the Chern-Simons term, from the phenomenological point of view, in principle, there is nothing to do with anomalies in Ward Identities. In fact, only the value at the zero axial vertex momenta is required and on this

kinematic point there is no violation in Ward Identities. When the anomalous term is added to the $AVV$ amplitude the $U(1)$ gauge invariance is present in all kinematic point that is the point of view adopted in the construction of the Standard Model by anomaly cancellation. If we follow this argumentation we must to import, when the $\Pi_b^{\mu\nu}$ calculation is in order, the anomalous term. [2]

We have finally arrived at the position where the answer to the question involved in this section can be furnished: the redefinition imposed by gauge invariance in the $AVV$ amplitude generates a subtraction that cancels exactly the remaining contribution eq.(37) to the value for the Chern-Simons term as previously conjectured in the literature [5]

## V. SUMMARY AND DISCUSSIONS

In this work we studied an extended version of Q.E.D. by the addition of an axial-vector term in Lagrangian ($\bar{\Psi}\not{A}\not{b}\gamma_5\Psi$) in order to exploit the possibility of Lorentz and CPT symmetry violation induced by radiative corrections (to one-loop order). The relevant quantity to analyze is the vacuum polarization tensor $\Pi_{\mu\nu}$ which could be decomposed in three parts: one is structurally identical to the pure Q.E.D. vacuum polarization tensor $\Pi_0^{\mu\nu}$ and the others are $\Pi_b^{\mu\nu}$ and $\Pi_{bb}^{\mu\nu}$ linear and quadratic in $b_\mu$ respectively. $\Pi_b^{\mu\nu}$ has the same mathematical form (closely related) as the famous $AVV$ triangle and $\Pi_{bb}^{\mu\nu}$ does not contribute [7].

Throughout the calculations we have not assumed any specific regularization scheme (or something equivalent) but instead we followed a simple strategy in dealing with the divergences: the integrals are assumed to be implicitly regulated by a function which is even in the integration variable and possesses a well defined connection limit. Under this assumption we could, by means of algebraic manipulation in the integrand, separate the finite content from the divergences. The latter were then displayed in terms of $I_{log}(m^2), I_{quad}(m^2)$ and differences between divergent integrals of same degree of divergences $\square, \nabla$ and $\Delta$ without any further assumption upon them. At this point we could readily map our procedure on the conventional ones (Dimensional Regularization, covariant Paulli-Villars, ...). This could be done by expressing $I_{log}(m^2)$ and $I_{quad}(m^2)$ according to the rules of the regularization scheme adopted and explicitly evaluating the differences $\square, \nabla$ and $\Delta$. In doing so we can distinguish two classes of regularizations depending on the value of those differences being zero or not. For instance in D.R. it can be shown that $\square, \nabla$ and $\Delta$ vanish. In schemes like that adopted in the ref.[6] these differences can be identified as their surface terms.

Having set the grounds for our analysis, in order to verify if an induced Chern-Simons term could be generated radiatively we proceeded to a consistent treatment for the vacuum polarization tensor. To start with, we choose to demand the U(1) gauge invariance of $\Pi_0^{\mu\nu}$ (pure Q.E.D.). This in turn implied that $\square = \nabla = \Delta = 0$. Now it is important to realize that these objects appear in the expression for $\Pi_{\mu\nu}$ with coefficients which are *not* ambiguous i.e. dependent on the combination $k_1 - k_2$. However two of them $\square$ and $\Delta$ do have ambiguous coefficients ($k_1$ and $k_2$ in other combinations than differences). Therefore there is no possibility to find a particular routing of the internal momenta so as to maintain gauge invariance. Thus gauge invariance itself eliminates the ambiguous terms. In this sense the consistency conditions ($\square = \nabla = \Delta = 0$) produce the same result as in DR. With this result in mind we proceeded to calculate the second term $\Pi_b^{\mu\nu}$, which can be identified with a particular kinematical situation of the $AVV$ amplitude. The most general explicit form of this amplitude was obtained in such way that all possibilities are still present. From the result so obtained three clearly situations, corresponding to different values for the Chern-Simons term, can be identified.

**i** <u>Surface's terms evaluation</u>.

In our general result eq.(32) if we interpret the divergent content remaining, represented by the $\Delta_{\mu\nu}$ object, as a surface term

$$-\Delta_{\mu\nu} = S_{\mu\nu} = \left(\frac{i}{(4\pi)^2}\right)\left(\frac{1}{2}\right) g_{\mu\nu} \tag{53}$$

the value for the Chern-Simons term is given by

$$\Pi_{\mu\nu\lambda}(p) = \left(\frac{3}{8\pi^2}\right) \varepsilon_{\mu\nu\lambda\beta} p^\beta \tag{54}$$

which corresponds to that obtained by other authors. In particular, this is the point of view adopted for the problem in ref. [1]. Our criticism in respect to this result resides on the fact that there is a hard price to pay in adopting

[2] Note that for the QED extended implications, strictly speaking, this a choice and not a requirement. If however we take the QED extended as embedding in a more general theory then our specific choice became a requirement.

this way. All perturbative divergent amplitudes, with divergence degree higher than logarithmic, are assumed as ambiguous quantities and the symmetry relations may be violated as we have showed analyzing the $\Pi_0^{\mu\nu}$ term for the full expression of the present problem.

**ii** U(1) gauge invariance of the $\Pi_0^{\mu\nu}$.

Guided by the consistency requirements of the calculational method used to evaluate divergent amplitudes in a more general sense we have learned that we need to choose $\Delta_{\mu\nu} = 0$ (as well as $\Box_{\alpha\beta\mu\nu} = \nabla_{\mu\nu} = 0$). With this interpretation all the results obtained by our calculational approach can be mapped in those corresponding to D.R. calculations in all places where this technique can be applied. In consequence the amplitudes will turn out free from the ambiguities associated to the arbitrary choices involved in the rotulation of the internal lines momenta in loops. In this context, the value for the Chern-Simons term so obtained is given by

$$\Pi_{\mu\nu\lambda}(p) = \left(\frac{1}{2\pi^2}\right)\varepsilon_{\mu\nu\lambda\beta}p^{\beta}, \tag{55}$$

which is, in consequence, ambiguities free. We can say that it is determined by the QED renormalizability consistency requirements in perturbative calculations.

**iii** Anomaly cancellation implications

In the expression obtained for $AVV$, once we eliminated the ambiguous term, we verified that the vector Ward identities were violated whereas the axial was obtained satisfied. *This is the same situation as we encounter in the pion decay where phenomenology tells us that an anomaly term must be added. This is done by choosing that U(1) gauge invariance is maintained. Thus we redefined $AVV$ by subtracting the anomalous term. Consequently we have a result that is gauge invariant for all momenta (not only at zero).*

In proceeding this way we adopted the same value for the mathematical structure involved in the present problem, the $AVV$ Green's function, as in the Standard Model construction by anomaly cancellation.

The anomalous term included exactly cancels the value obtained in $\Pi_{\mu\nu\lambda}(p)$ in eq.(37) which naturally establishes that $k_\mu$ vanishes according to the conjecture of the Colladay and Kostelecký [5],

$$\Pi_{\mu\nu\lambda}(p) = 0. \tag{56}$$

All we required to obtain our conclusion was $U(1)$ gauge invariance in addition to the procedure that we adopted to manipulate and compute the divergent quantities. In virtue of these conclusions we can give a safe answer to the question put on the title of this contribution: What is the role played by ambiguities in calculations of the radiatively induced Chern-Simons shift in extended Q.E.D.? Gauge invariance leaves no room for ambiguities in perturbative calculations. This conclusion, extracted from our analysis in the present problem, actually remains valid in all other contexts where perturbative calculations is applied.

This procedure can be used to treat any problem that involves divergences in perturbative calculations at any order in $\hbar$ (in Q.F.T.), for both abelian and non-abelian theories, as well as to calculate renormalization group coefficients [22]. The results obtained this way are ambiguity free. This framework yields a clear formulation of anomalies. The Ward identities are automatically satisfied where they should. Applications in the context of renormalizable and non-renormalizable theories have been successfully affected. The main advantage resides in the simplicity and clarity once no explicit form of regularization is needed. In other words divergent integrals are *not* explicitly calculated at any step of the calculations. All we need to require are general properties and the so called consistency conditions. Consequently if a regularization satisfies the consistency conditions it is thus not necessary.

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## APPENDIX A: GENERAL INTEGRALS FOR THE FINITE CONTENT OF ONE LOOP AMPLITUDES

**The Functions** $Z_k(p^2;m^2)$

We define

$$Z_k(p^2;m^2) = \int_0^1 dz\, z^k ln\left(\frac{p^2z(1-z)-m^2}{-m^2}\right), \tag{A1}$$

where $k$ is an integer, $m$ is the mass parameter which appears in the propagators, $p$ is some external momentum.

**The Functions** $\xi_{nm}$

When we consider one loop Feynman integrals associated to three point functions with two external momenta, the finite parts of the amplitudes are always related to the following general structures (same masses)

$$\xi_{nm}(p,q)=\int_0^1 dz\int_0^{1-z} dy\frac{z^n y^m}{Q(y,z)}, \tag{A2}$$

where

$$Q(y,z)=p^2y(1-y)+q^2z(1-z)-2(p\cdot q)yz-m^2. \tag{A3}$$

## APPENDIX B: DIVERGENT INTEGRALS

All the integrals below are divergent. We use the identity (8) to separate the divergent part of the finite part. Only the finite part of the integrals should depend on the external momenta. The remaining divergent integrals, now independent of the physical momenta, are organized in terms of a set of differences between divergent integrals eq.(9),(10),(11) and the basic divergent objects $I_{log}(m^2)$ and $I_{quad}(m^2)$. With this philosophy we show bellow some divergent integrals that are necessary to calculate the amplitude $\Pi_{\mu\nu}$. Thus

- $$\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+k_1)^2-m^2]}=I_{quad}(m^2)+k_{1\mu}k_{1\nu}\Delta_{\mu\nu} \tag{B1}$$
- $$\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]}=I_{log}(m^2)-\left(\frac{i}{(4\pi)^2}\right)Z_0((k_1-k_2)^2;m^2) \tag{B2}$$
- $$\begin{aligned}\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]}&=-2\,(k_1+k_2)_\alpha\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\alpha}{(k^2-m^2)^3}\\&+(k_1+k_2)_\mu\left(\frac{i}{(4\pi)^2}\right)Z_1((k_1-k_2)^2;m^2)\end{aligned} \tag{B3}$$
- $$\begin{aligned}&\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{[(k+k_1)^2-m^2][(k+k_2)^2-m^2]}=\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(k^2-m^2)^2}-\left(k_1^2+k_2^2\right)\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(k^2-m^2)^3}\\&+\left(k_{1\alpha}k_{1\beta}+k_{2\alpha}k_{2\beta}+k_{1\alpha}k_{2\beta}\right)\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{4k_\mu k_\nu k_\alpha k_\beta}{(k^2-m^2)^4}\\&+\left(\frac{i}{(4\pi)^2}\right)\Big\{-(k_1-k_2)_\mu(k_1-k_2)_\nu Z_2((k_1-k_2)^2;m^2)\\&\qquad-(k_1-k_2)^2g_{\mu\nu}\left[\frac{1}{4}Z_0((k_1-k_2)^2;m^2)-Z_2((k_1-k_2)^2;m^2)\right]\\&\qquad+k_{1\mu}\left[(k_1-k_2)_\nu Z_1((k_1-k_2)^2;m^2)\right]\\&\qquad+k_{1\nu}\left[(k_1-k_2)_\mu Z_1((k_1-k_2)^2;m^2)\right]\\&\qquad-k_{1\mu}k_{1\nu}Z_0((k_1-k_2)^2;m^2)\Big\}.\end{aligned}$$

- $$\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)[(p+k)^2-m^2][(q+k)^2-m^2]}=\left(\frac{i}{(4\pi)^2}\right)\xi_{00}(p,q) \tag{B4}$$
- $$\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu}{(k^2-m^2)[(p+k)^2-m^2][(q+k)^2-m^2]}=-\left(\frac{i}{(4\pi)^2}\right)\{q_\mu\xi_{10}(p,q)+p_\mu\xi_{01}(p,q)\} \tag{B5}$$
- $$\begin{aligned}&\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(k^2-m^2)[(p+k)^2-m^2][(q+k)^2-m^2]}=\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(k^2-m^2)^3}\\&-\left(\frac{i}{(4\pi)^2}\right)\left\{g_{\mu\nu}\left[\frac{1}{2}Z_0((p-q)^2;m^2)-\left(\frac{1}{2}+m^2\xi_{00}(p,q)\right)+\frac{q^2}{2}\xi_{10}(p,q)+\frac{p^2}{2}\xi_{01}(p,q)\right]\right.\\&\qquad\left.-p_\mu p_\nu\xi_{02}(p,q)-q_\mu q_\nu\xi_{20}(p,q)-(p_\mu q_\nu-p_\nu q_\mu)\xi_{11}(p,q)\right\}.\end{aligned} \tag{B6}$$

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